Runge Kutta Method 4th Order

Formula:

$$\frac{dy}{dx} = f(x_n, y_n), \qquad y = f(x)$$

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

$$x_{n+1} = x_n + h$$

Example: *Solve the difference equation* $\frac{dy}{dx} = y - x$ *(where* $y(0) = 2$*), find* $y(0.1)$ *and* $y(0.3)$ *correct to five decimal places using Range Kutta 4th order formula.*

Solution: *Given that* $y(0) = 2, \quad so, x_0 = 0$ *and* $y_0 = 2, \quad h = 0.1, \quad f(x_n, y_n) = y - x$

$$k_1 = hf(x_0, y_0) = (0.1) \times f(0, 2) = 0.2$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = (0.1) \times f(0.05, 2.1) = 0.205$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = (0.1) \times f(0.05, 2.1025) = 0.20525$$

$$k_4 = hf(x_0 + h, y_0 + k_3) = (0.1) \times f(0.1, 2.20525) = 0.21053$$

$$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 2 + \frac{1}{6}(0.2 + (2 \times 0.205) + (2 \times 0.20525) + 0.21053)$$

$$= 2.20517$$

Now $x_1 = 0.1$ *and* $y_1 = 2.20517$

$$k_1 = hf(x_1, y_1) = (0.1) \times f(0.1, 2.20517) = 0.21052$$

$$k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right) = (0.1) \times f(0.15, 2.31043) = 0.21604$$

$$k_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right) = (0.1) \times f(0.15, 2.31319) = 0.21632$$

$$k_4 = hf(x_1 + h, y_1 + k_3) = (0.1) \times f(0.2, 2.42149) = 0.22215$$

$$y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 2.20517 + \frac{1}{6}(0.21052 + (2 \times 0.21604) + (2 \times 0.21632) + 0.22215)$$

$$= 2.42140$$

$Now\ x_2 = 0.2\ and\ y_2 = 2.42140$

$k_1 = hf(x_2, y_2) = (0.1) \times f(0.2, 2.42140) = 0.22214$

$k_2 = hf\left(x_2 + \frac{h}{2}, y_2 + \frac{k_1}{2}\right) = (0.1) \times f(0.25, 2.53247) = 0.22825$

$k_3 = hf\left(x_2 + \frac{h}{2}, y_2 + \frac{k_2}{2}\right) = (0.1) \times f(0.25, 2.53553) = 0.22855$

$k_4 = hf(x_2 + h, y_2 + k_3) = (0.1) \times f(0.3, 2.64995) = 0.23500$

$$y_3 = y_2 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 2.42140 + \frac{1}{6}(0.22214 + (2 \times 0.22825) + (2 \times 0.22855) + 0.23500)$$

$$= 2.64986$$

$Therefore,\quad we\ get\ y(0.1) = 2.20517\ and\ y(0.3) = 2.64986$