Runge Kutta Method 2nd Order

Formula:

$$\frac{dy}{dx} = f(x_n, y_n), \qquad y = f(x)$$

$$y_{n+1} = y_n + \frac{h}{2}(k_1 + k_2)$$

$$k_1 = f(x_n, y_n)$$

$$k_2 = f(x_n + h, y_n + hk_1)$$

$$x_{n+1} = x_n + h$$

Example: *Solve the difference equation* $\frac{dy}{dx} = y - x$ *(where* $y(0) = 2$*), find* $y(0.1)$ *and* $y(0.3)$ *correct to five decimal places using Range Kutta 2nd order formula.*

Solution: *Given that* $y(0) = 2$, *so,* $x_0 = 0$ *and* $y_0 = 2$, $h = 0.1$, $f(x_n, y_n) = y - x$

$$k_1 = f(x_0, y_0) = f(0, 2) = 2$$

$$k_2 = f(x_0 + h, y_0 + hk_1) = f(0.1, 2.2) = 2.1$$

$$y_1 = y_0 + \frac{h}{2}(k_1 + k_2) = 2 + \frac{0.1}{2}(2 + 2.1) = 2.205$$

Now $x_1 = 0.1$ *and* $y_1 = 2.205$

$$k_1 = f(x_1, y_1) = f(0.1, 2.205) = 2.105$$

$$k_2 = f(x_1 + h, y_1 + hk_1) = f(0.2, 2.4155) = 2.2155$$

$$y_2 = y_1 + \frac{h}{2}(k_1 + k_2) = 2.205 + \frac{0.1}{2}(2.105 + 2.2155) = 2.42103$$

Now $x_2 = 0.2$ *and* $y_2 = 2.42103$

$$k_1 = f(x_2, y_2) = f(0.2, 2.42103) = 2.22103$$

$$k_2 = f(x_2 + h, y_2 + hk_1) = f(0.3, 2.64313) = 2.34313$$

$$y_3 = y_2 + \frac{h}{2}(k_1 + k_2) = 2.42103 + \frac{0.1}{2}(2.22103 + 2.34313) = 2.64923$$

Therefore, *we get* $y(0.1) = 2.205$ *and* $y(0.3) = 2.64923$