Newton Raphsan Method

$$Formula: \quad x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad for\ n = 0, 1, 2, \dots \dots \dots$$

$Example$: $Use\ Newton's\ method\ to\ find\ root\ of\ the\ equation\ x^3 - 2x - 5 = 0$

$Solution$: $f(x) = x^3 - 2x - 5 \quad and \quad f'(x) = 3x^2 - 2$

$f(1) = (1)^3 - (2 \times 1) - 5 = -6, \quad which\ is\ negative$
$f(2) = (2)^3 - (2 \times 2) - 5 = -1, \quad which\ is\ negative$
$f(3) = (3)^3 - (2 \times 3) - 5 = 16, \quad which\ is\ positive$
$So, the\ root\ lies\ between\ 2\ and\ 3$

$x_0 = 2$
$f(x_0) = (2)^3 - (2 \times 2) - 5 = -1$
$f'(x_0) = (3 \times 2^2) - 2 = 10$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{(-1)}{10} = 2.1$$

$f(x_1) = (2.1)^3 - (2 \times 2.1) - 5 = 0.061$
$f'(x_1) = (3 \times (2.1)^2) - 2 = 11.23$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.1 - \frac{0.061}{11.23} = 2.09456812$$

$f(x_2) = (2.09456812)^3 - (2 \times 2.09456812) - 5 = 0.000185710849$
$f'(x_2) = (3 \times (2.09456812)^2) - 2 = 11.1616468$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.09456812 - \frac{0.000185710849}{11.1616468} = 2.09455148$$

$f(x_3) = (2.09455148)^3 - (2 \times 2.09455148) - 5 = -0.00000002$
$f'(x_3) = (3 \times (2.09455148)^2) - 2 = 11.1614377$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 2.09455148 - \frac{(-0.00000002)}{11.1614377} = 2.09455148$$

$So, the\ root\ is\ 2.09455148$

n	x_n	$f(x_n) = x^3 - 2x - 5$	$f'(x_n) = 3x^2 - 2$	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
0	2	-1	10	2.1
1	2.1	0.061	11.23	2.09456812
2	2.09456812	0.000185710849	11.1616468	2.09455148
3	2.09455148	-0.00000002	11.1614377	2.09455148