$Newton's\ Forward\ Interpolation$

$Formula:$

$$g(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots + \frac{p(p-1)(p-2)\ldots(p-n-1)}{n!}\Delta^n y_0$$

$Here \quad p = \dfrac{x - x_0}{h}$

n	y_n	Δy_n	$\Delta^2 y_n$	$\Delta^3 y_n$	$\Delta^4 y_n$
0	$y_0 = f(x_0)$	$\Delta y_0 = y_1 - y_0$	$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$	$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$	$\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0$
1	$y_1 = f(x_1)$	$\Delta y_1 = y_2 - y_1$	$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$	$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$	
2	$y_2 = f(x_2)$	$\Delta y_2 = y_3 - y_2$	$\Delta^2 y_2 = \Delta y_3 - \Delta y_2$		
3	$y_3 = f(x_3)$	$\Delta y_3 = y_4 - y_3$			
4	$y_4 = f(x_4)$				

$Example:$ $Interpolate\ the\ value\ of\ the\ function\ corresponding\ to\ x = 9\ using\ Newton's\ forward\ interpolation\ formula\ from\ the\ following\ set\ of\ data$

x	4	6	8	10
$f(x)$	19	40	79	142

$Solution:$

$$p = \frac{9-4}{2} = 2.5$$

n	y_n	Δy_n	$\Delta^2 y_n$	$\Delta^3 y_n$
0	$y_0 = 19$	$\Delta y_0 = 40 - 19 = 21$	$\Delta^2 y_0 = 39 - 21 = 18$	$\Delta^3 y_0 = 24 - 18 = 6$
1	$y_1 = 40$	$\Delta y_1 = 79 - 40 = 39$	$\Delta^2 y_1 = 63 - 39 = 24$	
2	$y_2 = 79$	$\Delta y_2 = 142 - 79 = 63$		
3	$y_3 = 142$			

$$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$= 19 + [2.5 \times 21] + \left[\frac{\{2.5 \times (2.5-1)\}}{2!} \times 18\right] + \left[\frac{\{2.5 \times (2.5-1) \times (2.5-2)\}}{3!} \times 6\right]$$

$$= 19 + 52.5 + \left[\frac{\{2.5 \times 1.5\}}{2} \times 18\right] + \left[\frac{\{2.5 \times 1.5 \times 0.5\}}{6} \times 6\right]$$

$$= 19 + 52.5 + 33.75 + 1.875$$

$$= 107.125$$