Newton's Backward Interpolation

Formula:

$$g(x) = y_n + q\Delta y_n + \frac{q(q+1)}{2!}\Delta^2 y_n + \frac{q(q+1)(q+2)}{3!}\Delta^3 y_n + \frac{q(q+1)(q+2)(q+3)}{4!}\Delta^4 y_n + \cdots$$

Here $q = \frac{x - x_n}{h}$

n	y_n	Δy_n	$\Delta^2 y_n$	$\Delta^3 y_n$	$\Delta^4 y_n$
0	$y_0 = f(x_0)$	$\Delta y_0 = y_1 - y_0$	$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$	$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$	$\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0$
1	$y_1 = f(x_1)$	$\Delta y_1 = y_2 - y_1$	$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$	$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$	
2	$y_2 = f(x_2)$	$\Delta y_2 = y_3 - y_2$	$\Delta^2 y_2 = \Delta y_3 - \Delta y_2$		
3	$y_3 = f(x_3)$	$\Delta y_3 = y_4 - y_3$			
4	$y_4 = f(x_4)$				

Example: *Interpolate the value of the function corresponding to* $x = 9$ *using Newton's backward interpolation formula from the following set of data*

x	4	6	8	10
$f(x)$	19	40	79	142

Solution:

$$q = \frac{9 - 10}{2} = -0.5$$

n	y_n	Δy_n	$\Delta^2 y_n$	$\Delta^3 y_n$
0	$y_0 = 19$	$\Delta y_0 = 40 - 19 = 21$	$\Delta^2 y_0 = 39 - 21 = 18$	$\Delta^3 y_0 = 24 - 18 = 6$
1	$y_1 = 40$	$\Delta y_1 = 79 - 40 = 39$	$\Delta^2 y_1 = 63 - 39 = 24$	
2	$y_2 = 79$	$\Delta y_2 = 142 - 79 = 63$		
3	$y_3 = 142$			

$$f(x) = y_n + q\Delta y_n + \frac{q(q+1)}{2!}\Delta^2 y_n + \frac{q(q+1)(q+2)}{3!}\Delta^3 y_n$$

$$= 142 + [(-0.5) \times 63] + \left[\frac{\{(-0.5) \times (-0.5 + 1)\}}{2!} \times 24\right] + \left[\frac{\{(-0.5) \times (-0.5 + 1) \times (-0.5 + 2)\}}{3!} \times 6\right]$$

$$= 142 + [-31.5] + \left[\frac{\{(-0.5) \times (0.5)\}}{2} \times 24\right] + \left[\frac{\{(-0.5) \times (0.5) \times (1.5)\}}{6} \times 6\right]$$

$$= 142 - 31.5 - 3 - 0.375$$

$$= 107.125$$