Lagrange's Interpolation

Formula:

$$g(x)=\left\{\frac{(x-x_1)(x-x_2)(x-x_3)\dots\dots\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\dots\dots\dots(x_0-x_n)}\times f(x_0)\right\}$$
$$+\left\{\frac{(x-x_0)(x-x_2)(x-x_3)\dots\dots\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\dots\dots\dots(x_1-x_n)}\times f(x_1)\right\}$$
$$+\left\{\frac{(x-x_0)(x-x_1)(x-x_3)\dots\dots\dots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\dots\dots\dots(x_2-x_n)}\times f(x_2)\right\}$$
$$+\ \dots\dots\dots+\left\{\frac{(x-x_0)(x-x_1)(x-x_2)\dots\dots\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\dots\dots\dots(x_n-x_{n-1})}\times f(x_n)\right\}$$

Example: *Interpolate the value of the function corresponding to* $x=4$ *using Langrange's interpolation formula from the following set of data*

x	2	3	5	8
$f(x)$	10	15	25	40

Solution:

$$f(x)=\left\{\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}\times f(x_0)\right\}+\left\{\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}\times f(x_1)\right\}$$
$$+\left\{\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}\times f(x_2)\right\}+\left\{\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}\times f(x_3)\right\}$$
$$=\left\{\frac{(4-3)(4-5)(4-8)}{(2-3)(2-5)(2-8)}\times 10\right\}+\left\{\frac{(4-2)(4-5)(4-8)}{(3-2)(3-5)(3-8)}\times 15\right\}+\left\{\frac{(4-2)(4-3)(4-8)}{(5-2)(5-3)(5-8)}\times 25\right\}$$
$$+\left\{\frac{(4-2)(4-3)(4-5)}{(8-2)(8-3)(8-5)}\times 40\right\}$$
$$=\left\{\frac{(1)(-1)(-4)}{(-1)(-3)(-6)}\times 10\right\}+\left\{\frac{(2)(-1)(-4)}{(1)(-2)(-5)}\times 15\right\}+\left\{\frac{(2)(1)(-4)}{(3)(2)(-3)}\times 25\right\}+\left\{\frac{(2)(1)(-1)}{(6)(5)(3)}\times 40\right\}$$
$$=\left\{\frac{(4)}{(-18)}\times 10\right\}+\left\{\frac{(8)}{(10)}\times 15\right\}+\left\{\frac{(-8)}{(-18)}\times 25\right\}+\left\{\frac{(-2)}{(90)}\times 40\right\}$$
$$=\left\{-\frac{20}{9}\right\}+12+\frac{100}{9}+\left\{-\frac{8}{9}\right\}$$
$$=\frac{-20+108+100-8}{9}$$
$$=\frac{180}{9}$$
$$=20$$