Gauss Seidel

Formula :

$$a_{11}x_1 + a_{12}x_2 + \ \dots\dots\dots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ \dots\dots\dots + a_{2n}x_n = b_2$$
$$\dots \quad \dots \quad \dots \quad \dots \quad \dots$$
$$a_{n1}x_1 + a_{n2}x_2 + \ \dots\dots\dots + a_{nn}x_n = b_n$$

$$x_1^{k+1} = \frac{1}{a_{11}}\left(b_1 - a_{12}x_2^k - a_{13}x_3^k - a_{14}x_4^k - \ \dots\dots\dots - a_{1n}x_n^k\right)$$
$$x_2^{k+1} = \frac{1}{a_{22}}\left(b_2 - a_{21}x_1^{k+1} - a_{23}x_3^k - a_{24}x_4^k - \ \dots\dots\dots - a_{2n}x_n^k\right)$$
$$x_3^{k+1} = \frac{1}{a_{33}}\left(b_3 - a_{31}x_1^{k+1} - a_{32}x_2^{k+1} - a_{34}x_4^k - \ \dots\dots\dots - a_{3n}x_n^k\right)$$
$$\dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots$$
$$x_n^{k+1} = \frac{1}{a_{nn}}\left(b_n - a_{n2}x_2^{k+1} - a_{n3}x_3^{k+1} - a_{n3}x_3^{k+1} - \ \dots\dots\dots - a_{nn-1}x_{n-1}^{k+1}\right)$$

Consider 3 equation to understand gauss seidel method

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$x_1^{k+1} = \frac{b_1 - a_{12}x_2^k - a_{13}x_3^k}{a_{11}}$$
$$x_2^{k+1} = \frac{b_2 - a_{21}x_1^{k+1} - a_{23}x_3^k}{a_{22}}$$
$$x_3^{k+1} = \frac{b_3 - a_{31}x_1^{k+1} - a_{32}x_2^{k+1}}{a_{33}}$$

Consider 4 equation to understand gauss seidel method

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 = b_3$$
$$a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = b_4$$

$$x_1^{k+1} = \frac{b_1 - a_{12}x_2^k - a_{13}x_3^k - a_{14}x_4^k}{a_{11}}$$
$$x_2^{k+1} = \frac{b_2 - a_{21}x_1^{k+1} - a_{23}x_3^k - a_{24}x_4^k}{a_{22}}$$
$$x_3^{k+1} = \frac{b_3 - a_{31}x_1^{k+1} - a_{32}x_2^{k+1} - a_{34}x_4^k}{a_{33}}$$
$$x_4^{k+1} = \frac{b_4 - a_{41}x_1^{k+1} - a_{42}x_2^{k+1} - a_{43}x_3^{k+1}}{a_{44}}$$

$Example:$ $Use\ Gauss\ Jacobi\ to\ solve$

$$5x - 2y + 3z = -1$$
$$-3x + 9y + z = 2$$
$$2x - y - 7z = 3$$

$Solution:$ $Let\ k = 0,$ $Assume\ that\ (x^0, y^0, z^0) = (0, 0, 0)$

$$x^1 = \frac{b_1 - a_{12}y^0 - a_{13}z^0}{a_{11}} = \frac{(-1) - (-2 \times 0) - (3 \times 0)}{5} = -0.2$$

$$y^1 = \frac{b_2 - a_{21}x^2 - a_{23}z^0}{a_{22}} = \frac{2 - \big((-3) \times (-0.2)\big) - (1 \times 0)}{9} = 0.15556$$

$$z^1 = \frac{b_1 - a_{31}x^2 - a_{32}y^2}{a_{33}} = \frac{3 - \big(2 \times (-0.2)\big) - (-1 \times 0.15556)}{-7} = -0.50794$$

$$x^2 = \frac{b_1 - a_{12}y^1 - a_{13}z^1}{a_{11}} = \frac{(-1) - (-2 \times 0.15556) - \big(3 \times (-0.50794)\big)}{5} = 0.19090$$

$$y^2 = \frac{b_2 - a_{21}x^2 - a_{23}z^1}{a_{22}} = \frac{2 - \big((-3) \times (0.19090)\big) - \big(1 \times (-0.50794)\big)}{9} = 0.34229$$

$$z^2 = \frac{b_1 - a_{31}x^2 - a_{32}y^2}{a_{33}} = \frac{3 - \big(2 \times (0.19090)\big) - (-1 \times 0.34229)}{-7} = -0.42293$$

$$x^3 = \frac{b_1 - a_{12}y^2 - a_{13}z^2}{a_{11}} = \frac{(-1) - (-2 \times 0.34229) - \big(3 \times (-0.42293)\big)}{5} = 0.19067$$

$$y^3 = \frac{b_2 - a_{21}x^3 - a_{23}z^2}{a_{22}} = \frac{2 - (-3 \times 0.19067) - \big(1 \times (-0.42293)\big)}{9} = 0.33277$$

$$z^3 = \frac{b_1 - a_{31}x^3 - a_{32}y^3}{a_{33}} = \frac{3 - (2 \times 0.19067) - (-1 \times 0.33277)}{-7} = -0.42163$$

$$x^4 = \frac{b_1 - a_{12}y^3 - a_{13}z^3}{a_{11}} = \frac{(-1) - (-2 \times 0.33277) - \big(3 \times (-0.42163)\big)}{5} = 0.18609$$

$$y^4 = \frac{b_2 - a_{21}x^4 - a_{23}z^3}{a_{22}} = \frac{2 - (-3 \times 0.18609) - \big(1 \times (-0.42163)\big)}{9} = 0.33110$$

$$z^4 = \frac{b_1 - a_{31}x^4 - a_{32}y^4}{a_{33}} = \frac{3 - (2 \times 0.18609) - (-1 \times 0.33110)}{-7} = -0.42270$$

k	1	2	3	4
x^k	-0.2	0.19090	0.19067	0.18609
y^k	0.15556	0.34229	0.33277	0.33110
z^k	-0.50794	-0.42293	-0.42163	-0.42270