

Gauss Jacobi

Formula :

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

$$\dots \dots \dots$$
$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$x_1^{k+1} = \frac{1}{a_{11}} (b_1 - a_{12}x_2^k - a_{13}x_3^k - a_{14}x_4^k - \dots - a_{1n}x_n^k)$$

$$x_2^{k+1} = \frac{1}{a_{22}} (b_2 - a_{21}x_1^k - a_{23}x_3^k - a_{24}x_4^k - \dots - a_{2n}x_n^k)$$

$$x_3^{k+1} = \frac{1}{a_{33}} (b_3 - a_{31}x_1^k - a_{32}x_2^k - a_{34}x_4^k - \dots - a_{3n}x_n^k)$$

$$\dots \dots \dots$$

$$x_n^{k+1} = \frac{1}{a_{nn}} (b_n - a_{n2}x_2^k - a_{n3}x_3^k - a_{n3}x_3^k - \dots - a_{nn-1}x_{n-1}^k)$$

Consider 3 equation to understand gauss jacobi method

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$x_1^{k+1} = \frac{b_1 - a_{12}x_2^k - a_{13}x_3^k}{a_{11}}$$

$$x_2^{k+1} = \frac{b_2 - a_{21}x_1^k - a_{23}x_3^k}{a_{22}}$$

$$x_3^{k+1} = \frac{b_3 - a_{31}x_1^k - a_{32}x_2^k}{a_{33}}$$

Consider 4 equation to understand gauss jacobi method

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 = b_3$$

$$a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = b_4$$

$$x_1^{k+1} = \frac{b_1 - a_{12}x_2^k - a_{13}x_3^k - a_{14}x_4^k}{a_{11}}$$

$$x_2^{k+1} = \frac{b_2 - a_{21}x_1^k - a_{23}x_3^k - a_{24}x_4^k}{a_{22}}$$

$$x_3^{k+1} = \frac{b_3 - a_{31}x_1^k - a_{32}x_2^k - a_{34}x_4^k}{a_{33}}$$

$$x_4^{k+1} = \frac{b_4 - a_{41}x_1^k - a_{42}x_2^k - a_{43}x_3^k}{a_{44}}$$

Example : Use Gauss Jacobi to solve

$$5x - 2y + 3z = -1$$

$$-3x + 9y + z = 2$$

$$2x - y - 7z = 3$$

Solution : Let $k = 0$, Assume that $(x^0, y^0, z^0) = (0, 0, 0)$

$$x^1 = \frac{b_1 - a_{12}y^0 - a_{13}z^0}{a_{11}} = \frac{(-1) - (-2 \times 0) - (3 \times 0)}{5} = -0.2$$

$$y^1 = \frac{b_2 - a_{21}x^0 - a_{23}z^0}{a_{22}} = \frac{2 - (-3 \times 0) - (1 \times 0)}{9} = 0.22222$$

$$z^1 = \frac{b_3 - a_{31}x^0 - a_{32}y^0}{a_{33}} = \frac{3 - (2 \times 0) - (-1 \times 0)}{-7} = -0.42857$$

$$x^2 = \frac{b_1 - a_{12}y^1 - a_{13}z^1}{a_{11}} = \frac{(-1) - (-2 \times 0.22222) - (3 \times (-0.42857))}{5} = 0.14603$$

$$y^2 = \frac{b_2 - a_{21}x^1 - a_{23}z^1}{a_{22}} = \frac{2 - ((-3) \times (-0.2)) - (1 \times (-0.42857))}{9} = 0.20317$$

$$z^2 = \frac{b_3 - a_{31}x^1 - a_{32}y^1}{a_{33}} = \frac{3 - (2 \times (-0.2)) - (-1 \times 0.22222)}{-7} = -0.51746$$

$$x^3 = \frac{b_1 - a_{12}y^2 - a_{13}z^2}{a_{11}} = \frac{(-1) - (-2 \times 0.20317) - (3 \times (-0.51746))}{5} = 0.19174$$

$$y^3 = \frac{b_2 - a_{21}x^2 - a_{23}z^2}{a_{22}} = \frac{2 - (-3 \times 0.14603) - (1 \times (-0.51746))}{9} = 0.32839$$

$$z^3 = \frac{b_3 - a_{31}x^2 - a_{32}y^2}{a_{33}} = \frac{3 - (2 \times 0.14603) - (-1 \times 0.20317)}{-7} = -0.41587$$

$$x^4 = \frac{b_1 - a_{12}y^3 - a_{13}z^3}{a_{11}} = \frac{(-1) - (-2 \times 0.32839) - (3 \times (-0.41587))}{5} = 0.18088$$

$$y^4 = \frac{b_2 - a_{21}x^3 - a_{23}z^3}{a_{22}} = \frac{2 - (-3 \times 0.19174) - (1 \times (-0.41587))}{9} = 0.33234$$

$$z^4 = \frac{b_3 - a_{31}x^3 - a_{32}y^3}{a_{33}} = \frac{3 - (2 \times 0.19174) - (-1 \times 0.32839)}{-7} = -0.42070$$

k	1	2	3	4
x^k	-0.2	0.14603	0.19174	0.18088
y^k	0.22222	0.20317	0.32839	0.33234
z^k	-0.42857	-0.51746	-0.41587	-0.42070