Bisection Method

Intermediate Value Theorem :

This theorem states that if "f" is a continuous function on $[a, b]$ *and the sign of* $f(a)$ *is different from the sign of* $f(b)$*, that is* $f(a).f(b) < 0$*, then there exists a point c, in the intervel* (a, b) *such that* $f(c) = 0$.

The bisection method is defined as follows :

1. *Find an interval* (a, b) *in which in root lies, using intermediate value theorem.*
2. *Let* $c = (a + b)/2$. *If* $f(c) = 0$, *then* $x = c$ *is the root and the root is determined. Otherwise, use the intermediate value theorem to decide whether the root lies in* (a, c) *or* (b, c).

Example : *How many iterations of bisection method are required to be performed, to obtain smallest positive root of* $x^3 - 2x - 5 = 0$ *correct upto 2 decimal places.*

Solution : $f(x) = x^3 - 2x - 5$
$f(1) = (1)^3 - (2 \times 1) - 5 = -6$, *which is negative*
$f(2) = (2)^3 - (2 \times 2) - 5 = -1$, *which is negative*
$f(3) = (3)^3 - (2 \times 3) - 5 = 16$, *which is positive*
So, root of $x^3 - 2x - 5$ *lies in between 2 and 3, because* $f(2).f(3) < 0$

a	b	$c = (a + b)/2$	$f(c) = c^3 - (2 \times c) - 5$
2	3	$(2 + 3)/2 = 2.5$	$(2.5)^3 - (2 \times 2.5) - 5 = 5.625$
2	2.5	$(2 + 2.5)/2 = 2.25$	$(2.25)^3 - (2 \times 2.25) - 5 = 1.89063$
2	2.25	$(2 + 2.25)/2 = 2.125$	$(2.125)^3 - (2 \times 2.125) - 5 = 0.34570$
2	2.125	$(2 + 2.125)/2 = 2.0625$	$(2.0625)^3 - (2 \times 2.0625) - 5 = -0.35132$
2.0625	2.125	$(2.0625 + 2.125)/2 = 2.09375$	$(2.09375)^3 - (2 \times 2.09375) - 5 = -0.00894$
2.09375	2.125	$(2.09375 + 2.125)/2 = 2.109375$	$(2.109375)^3 - (2 \times 2.109375) - 5 = 0.16684$
2.09375	2.109375	$(2.09375 + 2.109375)/2 = 2.1015675$	$(2.1015675)^3 - (2 \times 2.1015675) - 5 = 0.07862$
2.09375	2.1015675	$(2.09375 + 2.1015675)/2 = 2.09765875$	$(2.09765875)^3 - (2 \times 2.09765875) - 5 = 0.03474$
2.09375	2.09765875	$(2.09375 + 2.09765875)/2 = 2.095704375$	$(2.095704375)^3 - (2 \times 2.095704375) - 5 = 0.01288$